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Change of the Independent Variable.

BY J. C. GLASHAN, Ottawa, Canada.

By Taylor's Theorem, if $u \equiv f(y)$ and $x \equiv \phi(y)$,

$$f(y + h) = u + hd_y u + h^2 \frac{d_y^2 u}{2!} + h^3 \frac{d_y^3 u}{3!} + \text{etc.}; \quad (\text{A})$$

also

$$\begin{aligned} f(y + h) &= f\phi^{-1}[x + \{\phi(y + h) - x\}] \\ &= u + \{\phi(y + h) - x\} d_x u + \{\phi(y + h) - x\}^2 \frac{d_x^2 u}{2!} + \{\phi(y + h) - x\}^3 \frac{d_x^3 u}{3!} + \text{etc.} \end{aligned} \quad (\text{B})$$

$$\phi(y + h) = x + hd_y x + h^2 \frac{d_y^2 x}{2!} + h^3 \frac{d_y^3 x}{3!} + \text{etc.} \quad (\text{C})$$

Substitute by (C) for $\{\phi(y + h) - x\}$ in (B), and equate coefficients of like powers of h in (A) and (B) thus reduced.

For convenience, let

$$\begin{aligned} x_1 &\equiv d_y x, & x_2 &\equiv \frac{d_y^2 x}{2!}, & x_3 &\equiv \frac{d_y^3 x}{3!}, \dots \\ u_1 &\equiv d_y u, & u_2 &\equiv \frac{d_y^2 u}{2!}, & u_3 &\equiv \frac{d_y^3 u}{3!}, \dots \end{aligned}$$

and $S_m^n \equiv$ the sum of the terms of weight m in the expansion of

$$(0_0 + x_1 + x_2 + x_3 + \dots)^n$$

(from which it immediately follows that, if $p > m$, $S_m^p = 0$ and that $S_m^m = x_1^m$).

$$u_1 = x_1 d_x u,$$

$$u_2 = x_2 d_x u + x_1^2 \frac{d_x^2 u}{2!},$$

$$u_3 = x_3 d_x u + 2x_1 x_2 \frac{d_x^2 u}{2!} + x_1^3 \frac{d_x^3 u}{3!},$$

$$u_4 = x_4 d_x u + (2x_1 x_3 + x_2^2) \frac{d_x^2 u}{2!} + 3x_1^2 x_2 \frac{d_x^3 u}{3!} + x_1^4 \frac{d_x^4 u}{4!},$$

$$u_5 = x_5 d_x u + 2(x_1 x_4 + x_2 x_3) \frac{d_x^2 u}{2!} + 3(x_1^2 x_3 + x_1 x_2^2) \frac{d_x^3 u}{3!} + 4x_1^3 x_2 \frac{d_x^4 u}{4!} + x_1^5 \frac{d_x^5 u}{5!},$$

and generally

$$u_n = S'_n \cdot d_x u + S_n^2 \cdot \frac{d_x^2 u}{2!} + S_n^3 \cdot \frac{d_x^3 u}{3!} + S_n^4 \cdot \frac{d_x^4 u}{4!} + \dots + S_n^n \cdot \frac{d_x^n u}{n!};$$

$$\therefore d_x u = \frac{u_1}{x_1}; \quad \frac{d_x^2 u}{2!} = \begin{vmatrix} x_1 & u_1 \\ x_2 & u_2 \\ x_1 \cdot x_1^2 & \end{vmatrix};$$

$$\frac{d_x^3 u}{3!} = \begin{vmatrix} x_1 & 0 & u_1 \\ x_2 & x_1^2 & u_2 \\ x_3 & 2x_1 x_2 & u_3 \\ x_1 \cdot x_1^2 \cdot x_1^3 & \end{vmatrix}; \quad \frac{d_x^4 u}{4!} = \begin{vmatrix} x_1 & 0 & 0 & u_1 \\ x_2 & x_1^2 & 0 & u_2 \\ x_3 & 2x_1 x_2 & x_1^3 & u_3 \\ x_4 & 2x_1 x_3 + x_2^2 & 3x_1^2 x_2 & u_4 \\ x_1 \cdot x_1^2 \cdot x_1^3 \cdot x_1^4 & \end{vmatrix};$$

$$\frac{d_x^5 u}{5!} = \begin{vmatrix} x_1 & 0 & 0 & 0 & u_1 \\ x_2 & x_1^2 & 0 & 0 & u_2 \\ x_3 & 2x_1 x_2 & x_1^3 & 0 & u_3 \\ x_4 & 2x_1 x_3 + x_2^2 & 3x_1^2 x_2 & x_1^4 & u_4 \\ x_5 & 2(x_1 x_4 + x_2 x_3) & 3(x_1^2 x_3 + x_1 x_2^2) & 4x_1^3 x_2 & u_5 \\ x_1 \cdot x_1^2 \cdot x_1^3 \cdot x_1^4 \cdot x_1^5 & \end{vmatrix};$$

and generally

$$\frac{d_x^n u}{n!} = \begin{vmatrix} S_1^1 & & & & u_1 \\ S_2^1 & S_2^2 & & & u_2 \\ S_3^1 & S_3^2 & S_3^3 & & u_3 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{n-1}^1 & S_{n-1}^2 & S_{n-1}^3 & S_{n-1}^{n-1} & u_{n-1} \\ S_n^1 & S_n^2 & S_n^3 & S_n^{n-1} & u_n \\ x_1 \cdot x_1^2 \cdot x_1^3 \cdot \dots \cdot x_1^n & \end{vmatrix},$$

which may be written

$$\frac{d_x^n u}{n!} = \{ S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_{n-1}^{n-1} \cdot u_n - S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_{n-2}^{n-2} \cdot u_{n-1} | S_n^{n-1} | + S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_{n-3}^{n-3} \cdot u_{n-2} | S_{n-1}^{n-2} \cdot S_n^{n-1} | \\ \dots (-)^{p-1} S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_{n-p}^{n-p} \cdot u_{n-p+1} | S_{n-p+2}^{n-p+1}, S_{n-p+3}^{n-p+2}, \dots S_n^{n-1} | + \dots \} \div S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_n^n$$

If $u = y$, this reduces to the last term, i. e., since in this case $u_1 = 1$, to

$$\frac{d_x^n u}{n!} = (-)^{n-1} | S_2^1, S_3^2, S_4^3, \dots, S_n^{n-1} | \div S_1^1 \cdot S_2^2 \cdot S_3^3 \dots S_n^n.$$